paper [1] appearing elsewhere in this issue. Here the author first assembles twelve criteria (1)–(12), of which the first eight are classical (such as results of Euler and others, and properties of $\sigma(\cdot)$); the ninth and tenth are proved in the present manuscript; the eleventh is due to Muskat. The twelfth is due to Hagis and McDaniel [2], also appearing in this issue.

The author then subdivides the set of odd perfect numbers n (if any) into cases (or subcases), repeatedly branching and drawing conclusions, until a lower bound $\geqq 10^{45}$ is derived in each case. Each such lower bound is a product (or a minimum of such products) of known factors and/or known underbounds for unknown necessary factors of every n (if any) in this case. The tools used are judiciously chosen in each case from the aforementioned criteria (1)–(12), results of Kanold and Norton, properties of $\sigma(\cdot)$, deductions from incomplete factorizations and about sources for 3's, etc.

The branching is done first on divisibility by various combinations of 3, 5, 7; then, primarily, on powers or groups of powers, first of 7 or 3, and then of other primes successively generated by $\sigma(\cdot)$ and branching. Some of the above tools are used to pre-exclude certain branches. At times, presumably for greater efficiency, this pattern is varied by the use of other branchings, such as on $11 \mid n$ versus $11 \nmid n$. A computer was used to find factors, generally the ones $< 10^5$, of the relevant $\sigma(p^\beta)$.

This case study is followed on page 47 by a useful outline which gives, for each case and subcase, its name, its defining restrictions, remarks (in some cases), and the deduced lower bound.

The supplement (pages 64–81), which raises the lower bound to 10^{50}, uses two additional tools, (14) due to Tuckerman and (15) due to Robbins and to Pomerance, together with further application of the previous methods.

The author's paper [1] includes 11 typical cases and subcases selected and edited from this manuscript. These illustrate most of the methods used; however, the specialist will want to consult the complete manuscript also.

BRYANT TUCKERMAN

IBM Thomas J. Watson Research Center
Yorktown Heights, New York 10598

1. PETER HAGIS, JR., "A lower bound for the set of odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 951-953.
2. PETER HAGIS, JR., & WAYNE L. MCDANIEL, "On the largest prime divisor of an odd perfect number," *Math. Comp.*, v. 27, 1973, pp. 955-957.

54 [10].—P. A. MORRIS, *A Catalogue of Trees*, University of the West Indies, St. Augustine, Trinidad, West Indies, October 1972. Ms. of 10 pp. + 46 computer sheets deposited in the UMT file.

This catalogue lists all unlabeled mathematical trees, without duplication, up to 13 nodes, inclusive. The trees are described by their node pairs, preceded by a code giving the number of edges; thus, for example, the tabular entry 04003, 0102, 0203, 0204 refers to the tree on 4 nodes with the 3 edges (1, 2), (2, 3), (2, 4).

AUTHOR'S SUMMARY